Standard Model

Relativistic Quantum Mechanics

QM & Special Relativity known and they
had to be combined to describe QED.

Klein-Gordon Equation

Describes spin 0 particles. (mesons)

$$(\Box + m^2)\psi = 0 \qquad \Box = \partial_\mu\partial^\mu = \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \vec{\nabla}^2$$

Density

$$\rho = i\hbar\left(\phi^*\dot{\phi} - \phi\dot{\phi}^*\right)$$

Current

$$J = -i\hbar c^2(\phi^*\vec{\nabla}\phi - \phi\vec{\nabla}\phi^*)$$

$$J_\mu J^\mu = 0 \qquad \frac{\partial\rho}{\partial t} + \vec{\nabla}\cdot\vec{J} = 0$$

Maxwell Equations already give massless KG equation:

$$\partial_\nu\partial^\nu A^\mu = 0$$

Pathological Problems

1. Density can be negative.
2. Extra degree of freedom is necessary due to 2nd derivative.
3. Single Particle Description not possible
4. Negative Energy Solutions and interactions

KG Eq.->Schrödinger Eq.

$$E \approx mc^2 + \frac{p^2}{2m}$$

$$\vec{p} \to -i\hbar\vec{\nabla} \qquad E \to i\hbar\frac{\partial}{\partial t}$$

Klein Paradox: E<V: Perfect Transmission

Klein-Gordon Field

$$\mathcal{L}_D = \frac{\dot{\phi}^2}{2} - \frac{\phi\prime^2}{2} - \frac{m^2\phi^2}{2}$$

Hamiltonian Density

$$\hat{\mathcal{H}} = \dot{\phi}\hat{\pi} - \hat{\mathcal{L}}$$

Fermi's Golden Rule

The probability of a quantum system transitioning from one state to another due to a perturbation changes over time.

$$\Gamma_{fi} = 2\pi|T_{fi}|^2\rho(E_i)$$

$$T_{fi} = \langle f|\hat{\mathcal{H}}'|i\rangle + \sum_{j\neq i}\frac{\langle f|\hat{\mathcal{H}}'|j\rangle\langle j|\hat{\mathcal{H}}'|i\rangle}{E_i - E_j} + \dots$$

dn is the number of accessible states in the energy range $E \to E + \mathrm{d}E$

$$\rho(E_i) = \left|\frac{\mathrm{d}n}{\mathrm{d}E}\right|_{E_i} = \int\frac{\mathrm{d}n}{\mathrm{d}E}\delta(E_i - E)\mathrm{d}E$$

KG equation applies to wave function is wrong.
KG describes the field operator. Not a single particle wave function.
Field operator has creation and annihilation parts.

Dirac Equation

Describes spin 1/2 particles. (fermions)

$$\vec{S}^2 = s(s+1) = \frac{1}{4}\vec{\Sigma} = \frac{3}{4}\mathrm{I}$$

$$(\not{p} - m)\psi = 0 \qquad (i\gamma^\mu\partial_\mu - m)\psi = 0$$

Relativistic corrections comes from Dirac eq.

$$\vec{\mu} = g_e\frac{e}{2mc}\vec{L} \qquad g_e = 2$$

$\gamma^5 \to$ *Helicity Operator for massless particles.*

Decomposition of the field into momentum modes, each of the modes behaves like a harmonic oscillator.

$$\hat{\phi}(x,t) = \int \mathrm{d}kN(k)\left[\underbrace{\hat{a}(k)e^{i(k.x-\omega t)}}_{annihilator} + \underbrace{\hat{a}^\dagger(k)e^{-i(kx-\omega t)}}_{creator}\right]$$

$$i\hbar\frac{\partial\psi}{\partial t} = \beta mc^2\psi - i\hbar c\vec{\alpha}\cdot\vec{\nabla}\psi = H_{Dirac}\psi$$

Dirac Field

$$\mathcal{L}_D = \bar{\psi}i\gamma^\mu\partial_\mu\psi - m\bar{\psi}\psi$$

Action:

$$S = \int\mathcal{L}_D\mathrm{d}t$$

Least Action: Lagrange-Euler Equation:

$$\delta S = 0 \Rightarrow \frac{\partial\mathcal{L}_D}{\partial\phi} - \partial^\mu\left(\frac{\partial\mathcal{L}_D}{\partial(\partial^\mu\phi)}\right) = 0$$

Equation of Motion:

$$(i\gamma^\mu\partial_\mu - m)\psi = 0$$

Momentum density

$$\pi = \frac{\partial\mathcal{L}_D}{\partial\dot{\psi}} = \bar{\psi}i\gamma^0 = i\psi^\dagger$$

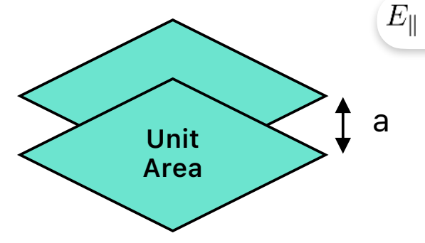
Hamiltonian Density:

$$\mathcal{H}_D = \pi\dot{\psi} - \mathcal{L}_D = -\bar{\psi}i\vec{\gamma}\cdot\vec{\nabla}\psi + m\bar{\psi}\psi$$

Casimir effect

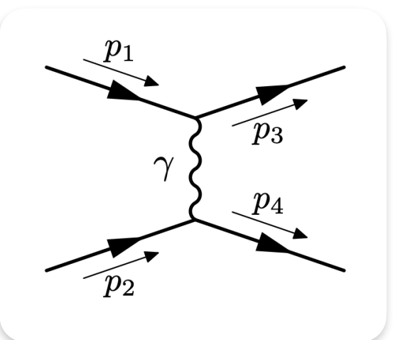
Attractive force between conducting infinite uncharged metal plates separated by a distance "a". About 1 nm = 1 atm pressure. For a finite volume vacuum expectation value is ∞.

$$E_\parallel = B_\perp = 0$$



$$P_C = \frac{F_C}{A} \approx -10^{-3}\,\mathrm{Pa}\cdot\left(\frac{\mu m}{L}\right)^4$$

Electromagnetic Interactions



$$M_{fi} = (\bar{u}_a\gamma^\mu u_a)\frac{ie_ae_b}{q^2}(\bar{u}_b\gamma_\mu u_b)$$

U(1) - Local Gauge Invariance

$$\mathcal{L}_D = i\bar{\psi}\gamma^\mu\partial_\mu\psi - m\bar{\psi}\psi$$

U(1) Phase Transformation:

$$\psi \to \psi' = e^{-ie\chi}\psi$$

$$\bar{\psi} \to \bar{\psi}' = e^{ie\chi}\bar{\psi}$$

Not Gauge Invariant!

$$\begin{aligned}\mathcal{L}_D' &= i[e^{ie\chi}\bar{\psi}]\gamma^\mu\partial_\mu[e^{-ie\chi}\psi] - m\bar{\psi}\psi\\ &= i[e^{ie\chi}\bar{\psi}]\gamma^\mu e^{-ie\chi}(\partial_\mu\psi - ie\partial_\mu\chi\psi) - m\bar{\psi}\psi\\ &\neq i\bar{\psi}\gamma^\mu\partial_\mu\psi - m\bar{\psi}\psi = \mathcal{L}_D\end{aligned}$$

$$A_\mu \to A_\mu' = A_\mu - \partial_\mu\chi(x)$$

$$\partial_\mu\psi(x) \longrightarrow \mathcal{D}_\mu\psi(x) = (\partial_\mu + ieA_\mu'(x))\psi(x)$$

$$\begin{aligned}\mathcal{L}_D' &= i\bar{\psi}'\gamma^\mu\mathcal{D}_\mu\psi' - m\bar{\psi}'\psi'\\ &= i\bar{\psi}'\gamma^\mu(\partial_\mu + ieA_\mu'(x))\psi' - m\bar{\psi}'\psi'\\ &= i\bar{\psi}'\gamma^\mu(\partial_\mu + ie[A_\mu(x) - \partial_\mu\chi(x)])\psi' - m\bar{\psi}\psi'\\ &= i\bar{\psi}e^{ie\chi}\gamma^\mu(\partial_\mu + ieA_\mu(x) - ie\partial_\mu\chi(x))\psi e^{-ie\chi} - m\bar{\psi}\psi\\ &= i\bar{\psi}\gamma^\mu(\partial_\mu\psi + ie\cancel{\partial_\mu}\chi\psi + ieA_\mu\psi - ie\cancel{\partial_\mu}\chi\psi) - m\bar{\psi}\psi\\ &= i\bar{\psi}\gamma^\mu(\partial_\mu + ieA_\mu(x))\psi - m\bar{\psi}\psi = \mathcal{L}_D\end{aligned}$$

Gauge Invariant! ✓

$$\begin{aligned}F_{\mu\nu}' &= \partial_\mu A_\nu' - \partial_\nu A_\mu'\\ &= \partial_\mu(A_\nu - \partial_\nu\chi) - \partial_\nu(A_\mu - \partial_\mu\chi)\\ &= (\partial_\mu\mathrm{A}_\nu - \partial_\nu\mathrm{A}_\mu) - (\partial_\mu\partial_\nu\chi - \partial_\nu\partial_\mu\chi) = F_{\mu\nu}\end{aligned}$$

$$\mathcal{L} = -\frac{1}{4}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} + i\bar{\psi}\not{\mathcal{D}}\psi - m\bar{\psi}\psi$$

SU(2) - Gauge Transformation

The mass term in the Dirac Lagrangian is:

$$\begin{aligned}m\bar{\psi}\psi &= \frac{1}{4}m\bar{\psi}(1-\gamma^5)(1-\gamma^5)\psi + \frac{1}{4}m\bar{\psi}(1+\gamma^5)(1+\gamma^5)\psi\\ &= m\bar{\psi}^R\psi^L + m\bar{\psi}^L\psi^R\end{aligned}$$

(Not Gauge Invariant!)

Fermions **can not have mass!**

CRIMINAL!

γ^5 → Parity Violator!

Standard Model Parameters

Parameters experimentally determined.
Couplings to gauge bosons: gs, e, sin_W -> 3 Parameters
CKM Parameters -> 4 Parameters
2Boson & 6Quark & 3Lepton Masses -> 11 Parameters
--+
18 free Standard Model Parameters
+8 mass of neutrinos and PMNS parameters
TOTAL 26 parameters Standard Model can be described.

Gauge Symmetry tells us formalism is redundant, it contains more in the equations, there is some freedom but it's not observable.

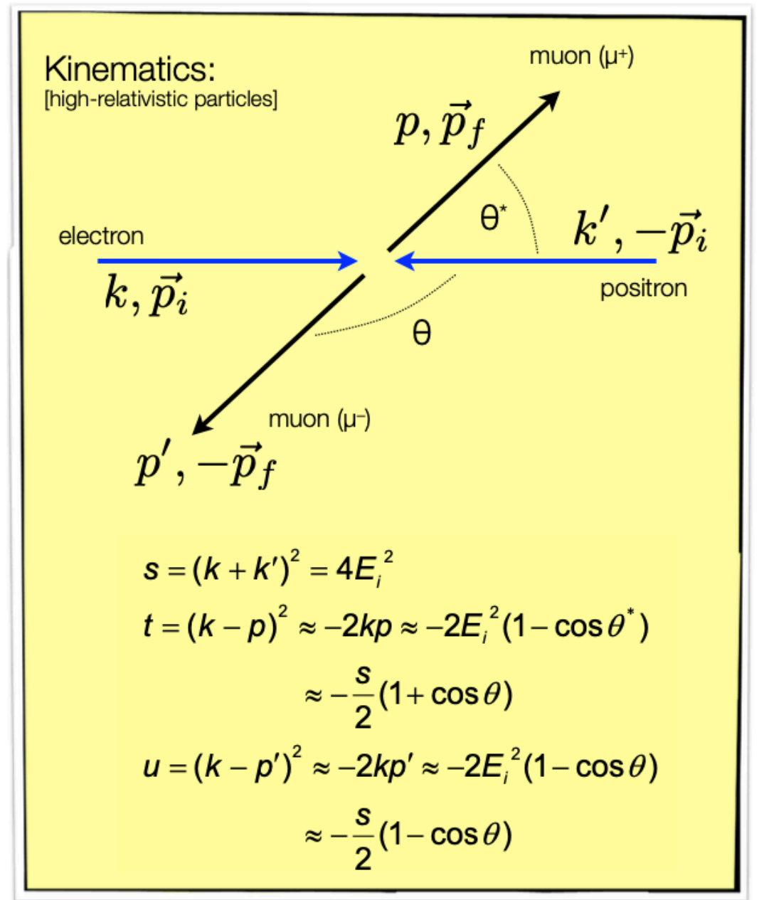
Muon Pair Production (Theory)

$$\overline{|M|^2}_{ee\to\mu\mu} = 2e^4\cdot\frac{t^2+u^2}{s^2}$$

$$\frac{d\sigma}{d\Omega} = \frac{e^4}{32\pi^2}\cdot\frac{1}{s}\cdot\frac{t^2+u^2}{s^2} = \frac{e^4}{64\pi^2}\cdot\frac{1}{s}\cdot(1+cos^2\theta)$$

with $e^2 = 4\pi\alpha$

$$\boxed{\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s}\cdot(1+cos^2\theta)}$$



$$s = (k+k')^2 = 4E_i^2$$
$$t = (k-p)^2 \approx -2kp \approx -2E_i^2(1-\cos\theta^*) \approx -\frac{s}{2}(1+\cos\theta)$$
$$u = (k-p')^2 \approx -2kp' \approx -2E_i^2(1-\cos\theta) \approx -\frac{s}{2}(1-\cos\theta)$$

Electroweak Interactions

Hyper-charge is the average charge of the weak isospin doublet.

$$\textbf{Weak Hypercharge} \to \mathrm{Y} = \mathrm{Q} - \mathrm{I}_3$$

$$I_3 = \begin{cases}\pm\frac{1}{2} & \text{for weak isospin doublet } I_w = \frac{1}{2}\\ 0 & \text{for singlet } I_w = 0\end{cases}$$

The weak isospin doublets:

$$\begin{pmatrix}\nu_e\\ e_L\end{pmatrix},\begin{pmatrix}\nu_\mu\\ \mu_L\end{pmatrix},\begin{pmatrix}\nu_\tau\\ \tau_L\end{pmatrix}$$

$$\begin{pmatrix}u_L\\ d_L\end{pmatrix},\begin{pmatrix}c_L\\ s_L\end{pmatrix},\begin{pmatrix}t_L\\ b_L\end{pmatrix}$$

+Quark mixing
Turn each other with **W** interactions.

$$\mathcal{L}_{EW} = -\frac{1}{4}G_j^{\mu\nu}G_{j\mu\nu} - \frac{1}{4}B^{\mu\nu}B_{\mu\nu} + i\bar{\psi}\gamma^\mu(\partial_\mu + igW_\mu T + ig'B_\mu Y)\psi$$

V-A Structure

Dirac Spinor is a 4-component wave function. Each component satisfies the Dirac and Klein Gordon equation. These components different than zero for particles at rest

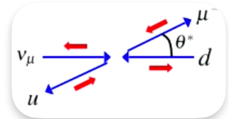
Particles $\psi = [\psi_1, \psi_2, \psi_3, \psi_4]^T$ Anti-particles

Chiral Projection Operators:

$$P_R = \frac{(1+\gamma^5)}{2} \qquad P_L = \frac{(1-\gamma^5)}{2}$$

Any spinor can be expressed as:

$$\Psi = P_R\Psi + P_L\Psi = \Psi_R + \Psi_L$$



spin=0 state no theta dependence!

U(1)-> Conservation of Electric Charge (Electromagnetic Interaction)

Photon, e-, n, p + charged particles

SU(2)-> Weak Nuclear Force (Weak interaction)

g, quarks, W, Z, mesons, neutrinos

SU(2)_L x U(1)_Y

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\bar{\psi}\not{D}\psi$$

$$eA_\mu = \frac{g_s}{2}\lambda_\nu G_\mu^\nu + \frac{g}{2}\vec{\tau}\vec{W}_\mu + \frac{g'}{2}YB_\mu$$

$$F_{\mu\nu}F^{\mu\nu} = G_{\mu\nu}G^{\mu\nu} + W_{\mu\nu}W^{\mu\nu} + B_{\mu\nu}B^{\mu\nu}$$

The Higgs Mechanism

Strong SU(3)_C x Weak SU(2)_L x Electromagnetic U(1)_Y

8 type of gluons, Color charge / Weak Isospin, W+,W-,Z / Hypercharge, Photon

$$\mathcal{L}_\phi = (\partial_\mu\phi^\dagger)(\partial^\mu\phi) - V(\phi) \qquad D_\mu = \partial_\mu + ieA_\mu$$

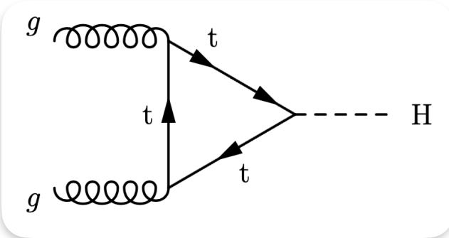
$$V(\phi) = -\mu^2|\phi^\dagger\phi| + \lambda|\phi^\dagger\phi|^2$$

$$\mathcal{L}_{Yuk} = c_f(\bar{\psi}_L\psi_R\phi + \bar{\psi}_R\psi_L\phi)$$

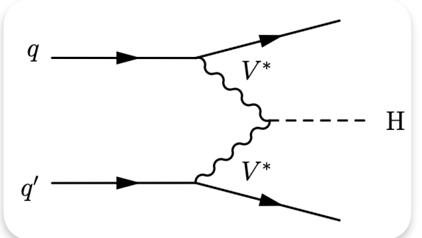
- Standard Model has only dimensionless couplings.

The Higgs Production Mechanisms

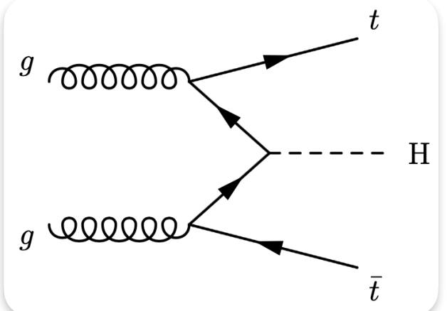
Gluon Fusion



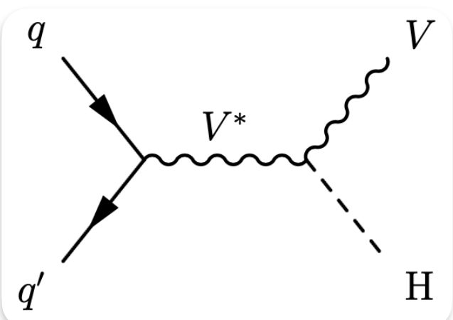
Vector Boson Fusion

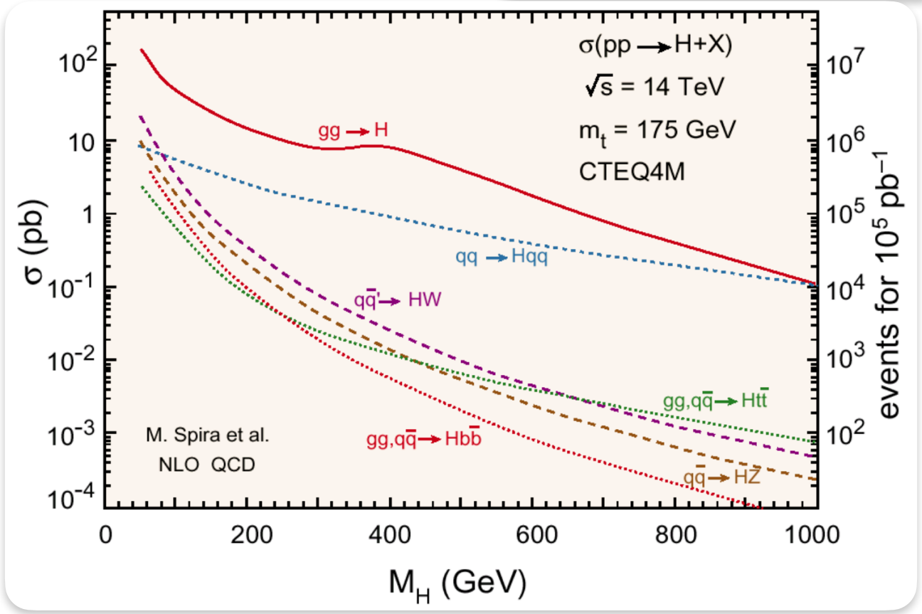


tt~Fusion



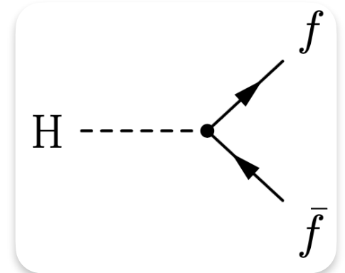
Associated



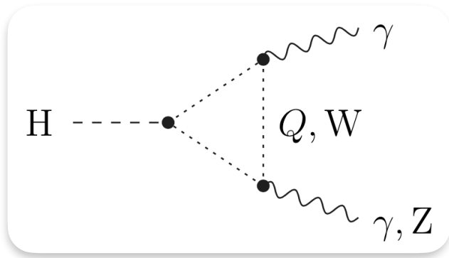


The Higgs Decay

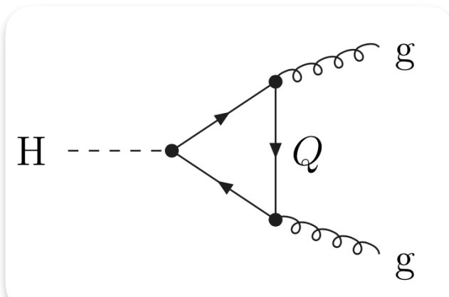
Yukawa Coupling



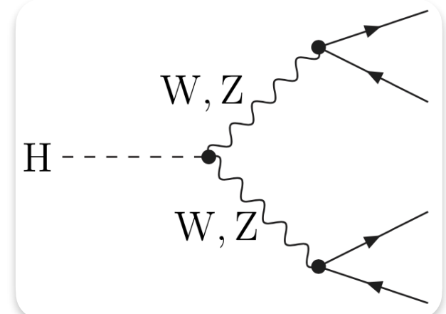
Photon Coupling

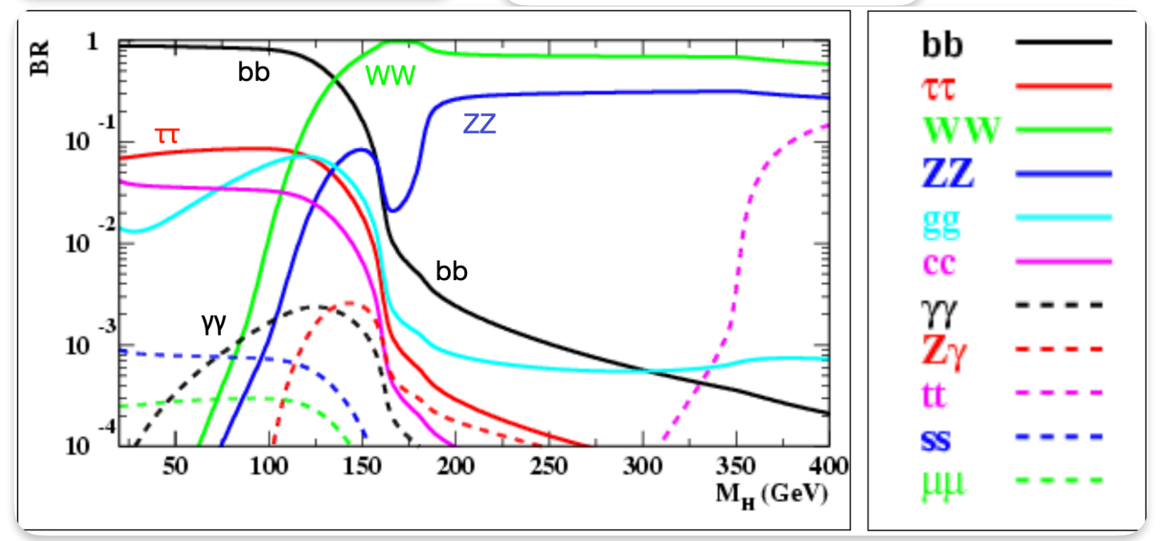


Gluon Coupling



Gauge Boson Coupling





Higgs couples to mass and b quark has the highest mass.
Coupling is proportional to mass and the heaviest particle
Higgs decay into b bbar.

Measurement of the "α"

Since 2019 Physical Constants has been changed.

$$\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c}$$

Exact!

Measurable!

Quantum Hall Effect:

$$R_k = \frac{h}{e^2} = \frac{\mu_0 c}{2\alpha_{QED}}$$

Electron g-factor:

$$\vec{\mu} = g_e\frac{e}{2mc}\vec{L} \qquad g_e = 2$$

Cyclotron

$$F = m\omega^2 r = evB = e\omega rB$$

Cyclotron Frequency = Larmor Frequency

$$\omega_c = \frac{e}{m}B = \omega_s \qquad a = \frac{\omega_s - \omega_c}{\omega_c}$$

$$a = \frac{\omega_s - \omega_c}{\omega_c}$$

$$\alpha_{QED}(\mu) = \frac{\alpha(m_e)}{1 - \frac{2\alpha(m_e)}{3\pi}\ln\left(\frac{\mu}{m_e}\right)}$$

Measurement of the Cross-section

#Events[Measured]

[Machine Parameter] Calculated

$$N = L\cdot\sigma$$

#bunches per beam

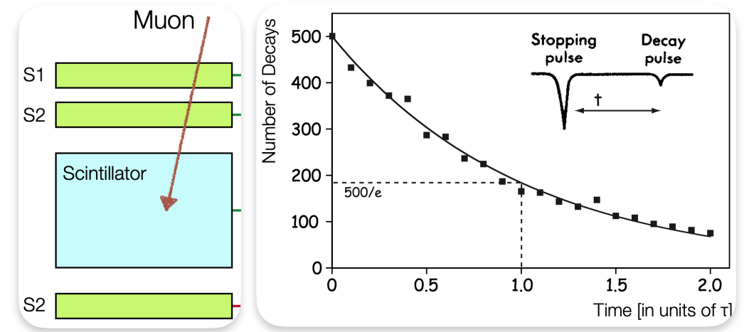
#particles per bunch

Revolution Frequency

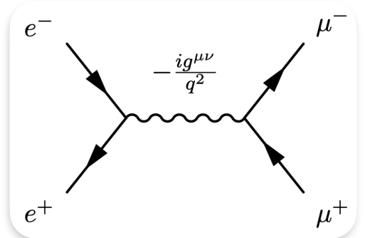
$$L = f\frac{nN_aN_b}{4\pi\sigma_x\sigma_y}$$

Beam Cross-section

Muon Lifetime Measurement:

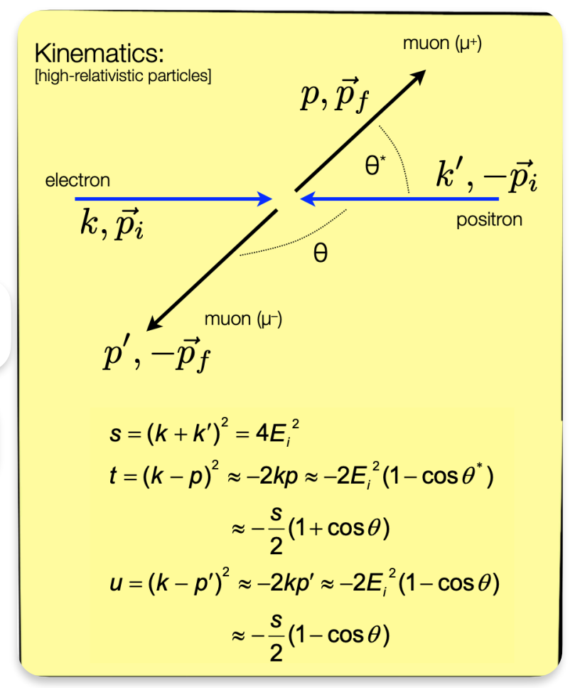


e+e- Scattering

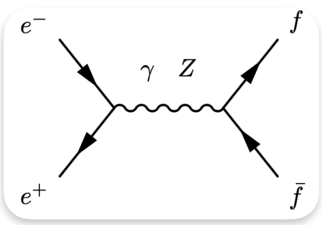


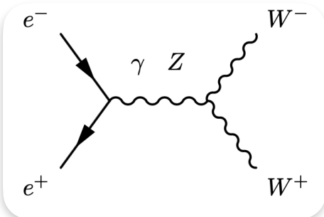
$$M_{fi} = \bar{\nu}_e\gamma_\mu u_e\left(-\frac{e^2}{q^2}\right)\bar{\nu}_\mu\gamma^\mu u_\mu$$

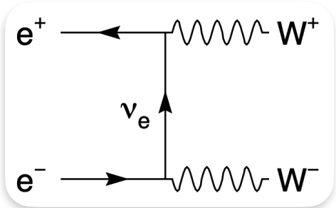
$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{1}{s\cdot 64\pi^2}\frac{|\vec{p}_f|}{|\vec{p}_i|}\overline{|M_{fi}|^2}$$

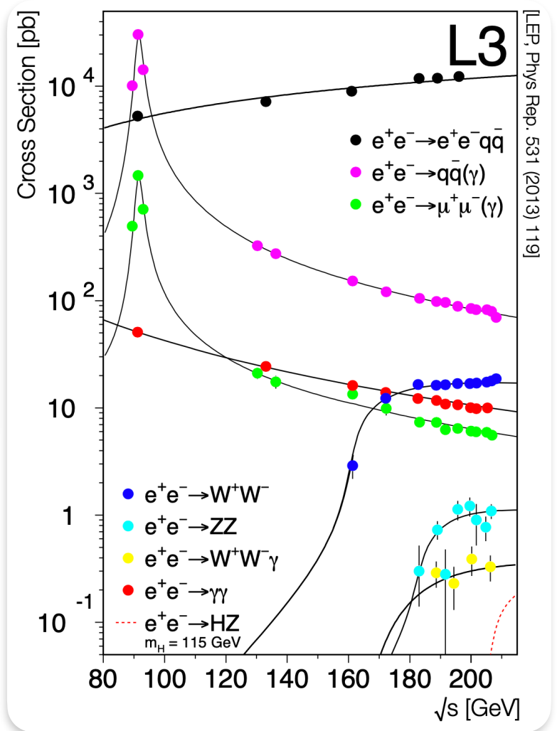


$$s = (k+k')^2 = 4E_i^2$$
$$t = (k-p)^2 \approx -2kp \approx -2E_i^2(1-\cos\theta^*) \approx -\frac{s}{2}(1+\cos\theta)$$
$$u = (k-p')^2 \approx -2kp' \approx -2E_i^2(1-\cos\theta) \approx -\frac{s}{2}(1-\cos\theta)$$

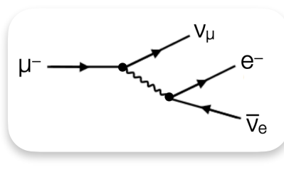






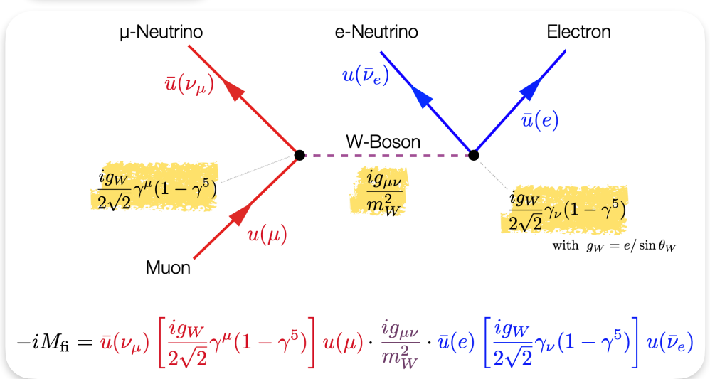


Muon Decay Process



$$M = [\bar{u}_\nu\gamma^\mu(1-\gamma^5)u_\mu]\frac{g_w^2}{8m_W^2}[\bar{u}_e\gamma^\mu(1-\gamma^5)u_\nu]$$

$$\Gamma_\mu = \frac{G_F^2 m_\mu^5}{192\pi^3}$$



$$-iM_{fi} = \bar{u}(\nu_\mu)\left[\frac{ig_W}{2\sqrt{2}}\gamma^\mu(1-\gamma^5)\right]u(\mu)\cdot\frac{ig_{\mu\nu}}{m_W^2}\cdot\bar{u}(e)\left[\frac{ig_W}{2\sqrt{2}}\gamma_\nu(1-\gamma^5)\right]u(\bar{\nu}_e)$$

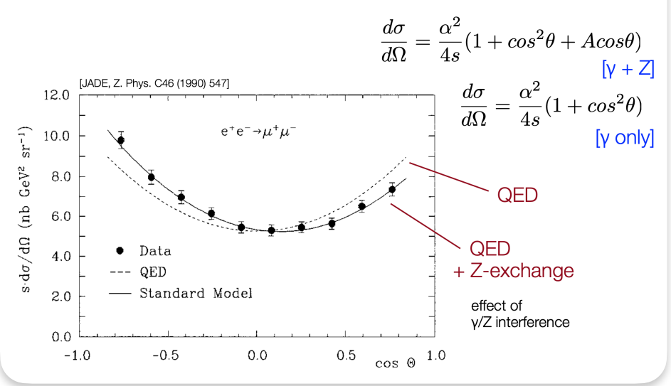
Muon Pair Production

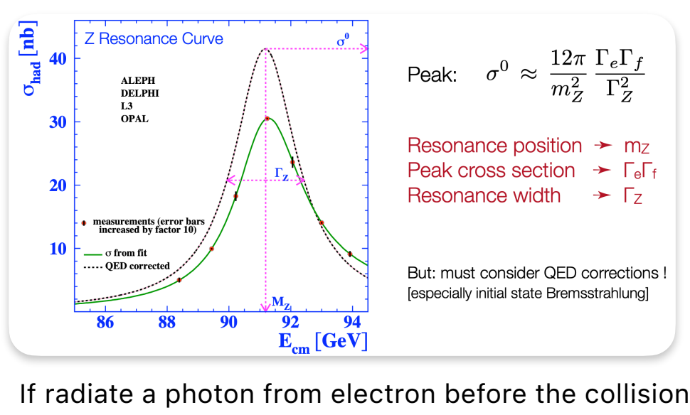
$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s}(1+cos^2\theta + Acos\theta)$$

[γ + Z]

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s}(1+cos^2\theta)$$

[γ only]





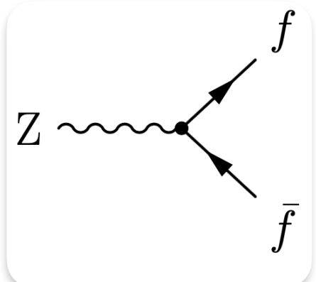
Peak: $\sigma^0 \approx \frac{12\pi}{m_Z^2}\frac{\Gamma_e\Gamma_f}{\Gamma_Z^2}$

Resonance position → m_Z
Peak cross section → $\Gamma_e\Gamma_f$
Resonance width → Γ_Z

But: must consider QED corrections !
[especially initial state Bremsstrahlung]

If radiate a photon from electron before the collision, they lose energy and can not produce Z boson anymore. I reduces the center of mass energy. Consequently cross-section starts to decrease.

Z Boson Decay



$$iA = \epsilon_\mu^\nu(k)\bar{u}_s(p)\left(-\frac{ig}{\cos\theta_W}\gamma^\mu\frac{1}{2}(v_f - a_f\gamma^5)\right)v_s'(p')$$

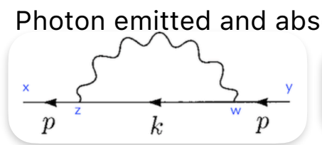
Lepton Universality: All leptons have same gv-ga universal couplings.

Sources of CP Violation

1. Complex CKM matrix, for neutrinos PMNS matrix
2. QCD Strong Interaction CP Violation problem(?)
3. Complex Phases in Higgs Potential

Renormalization

Photon emitted and absorbed instantaneously



$$\int_0^\infty \mathrm{d}^4k\frac{k}{k^2(p-k)^2} \to \textbf{Divergent!}$$

$$\int \mathrm{d}kk^{D-4}$$

Superficial degree of divergence

Finite degree of divergences can be absorbed in redefinitions of coupling constants and masses.

QCD

Parton->Coloured Quarks + Gluons

Advantages of using rapidity instead of the scattering angle: Rapidity is boost invariant.

Scaling Violations arises from assumption of quark dominated proton structure.

F(x,Q^2) :x↑

Probability of transmission depends on the weak mixing angle, Energy difference that contains the mass difference